NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: HONOURS IN STATISTICS	
QUALIFICATION CODE: 08BSOC	**LEVEL:** 8
COURSE CODE: STP801S	**COURSE NAME:** STOCHASTIC PROCESSES
SESSION: JUNE 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
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MODERATOR:	PROF L. KAZEMBE

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided. 2. Show clearly all the steps used in the calculations. 3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Question 1 [25 Marks]

1.1 Define

1.1.1 A an algebra $\mathcal{A}$ of a subset of X. (3)

1.1.2 Martingale process. (4)

1.1.3 A filtration $\{\mathrm{F}_t\}$. (4)

1.2 Let $\Omega = \{\omega_1, \omega_2\ , \omega_{3,}, \omega_4\}$, give an example of a filtration on Ω. (5)

1.3 Let $X_{n,}\quad n = 0,1,2$ ....be a stochastic Process in discrete time with a finite state space. State the conditions for X_n to be a Markov chain with stationary transition probability. (7)

1.4 Define a probability space $(\Omega, \Sigma, \mathbb{p}\ )$. (2)

Question 2 [25 Marks]

2.1 Define an absorbing state of a Markov chain. (2)

2.2 Find the long-term trend for the transition matrix given by $\begin{array}{c} \quad 1 \quad 2 \quad 3 \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} .3 & .2 & .5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$. (12)

2.3 When does a stochastic process becomes a simple Random Walk? (3)

2.4 Let $\{\chi_n\}_{n\epsilon N_0}$ be a simple random walk with parameter p. Prove that the distribution of the random variable χ_n given by the generating function $\mathrm{P}_x(s) = \sum_{k=0}^{\infty} P_k S^k$ of a poison function is $e^{\lambda(s-1)}$. (8)

Question 3 [25 Marks]

3.1 Define a discrete time Markov Chain with transition matrix $p(i,j)$. (5)

3.2 Suppose that in the Gambler's ruin chain, the transition probability has $p(i, i+1) = 0.4$, $p(i, i-1) = 0.6$, if $0 < i < N, p(0,0) = 1, p(N,N) = 1$ and $N = 5$.

Find the transition matrix. (4)

3.3 Let $\{\chi_n\}_{n\epsilon N_0}$ be a simple random walk with parameter p. Prove that the distribution of the random variable χ_n given by the generating function $\mathrm{P}_x(s) = \sum_{k=0}^{\infty} P_k S^k$ of a geometric function is $\frac{p}{1-qs}$. (3)

3.4 A transition matrix $p = \begin{bmatrix} .65 & .28 & .07 \\ .15 & .67 & .18 \\ .12 & .36 & .52 \end{bmatrix}$ shows the probability of a change in income class from one generation to the next, with $p_{i,j}$ representing the probability of changing from sate i to state j in general. Use p^k , when k =2 or 3 to solve the following:

3.4.1 Find the probability that a parent in state 1 (Lower class) will have a grandchild in state 3 (Upper class). (6)

3.4.2 Use matrix manipulation to show that a person in state 2 (middle class) will have a great grandchild in state 2 (middle class). (7)

Question 4 [25 Marks]

4.1 Define a regular Markov chain (2)

4.2 Show that for larger values of n, and transition P,

$vP^n \approx V$ where v is a vector (9)

4.3 Find the long-range trend for the Markov Chain in the income class with a transition matrix

Next Generation

Current state $\begin{bmatrix} .65 & .28 & .07 \\ .15 & .67 & .18 \\ .12 & .36 & .52 \end{bmatrix}$. (14)

END of EXAM